NUMERICAL SIMULATION OF THE INSTABILITY OF AN ION BEAM

S. G. Alikhanov and S. L. Musher

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We consider a numerical model of the instability of an ion beam. The energy of the oscillations is concentrated mainly in several neighboring modes that have a maximum increment in the linear theory. The instability has been traced up to the complete vanishing of the beam.

We study in the present paper, using the method of particles in cells, the instability of an ion beam in a non-isothermal plasma $T_e \gg T_i$. It is known [1-3] that in this case ion-acoustic oscillations are excited. The nonlinear stage of development of the instability calls for a kinetic analysis, and this indeed is the reason for using the method of particles in cells (see also [4,5]). A similar method was used in [4] to investigate the instability of an electron beam against a background of immobile cold ions, and in [6] to study the entry of beam into a plasma; the instability of two identical mutually interpenetrating ion beams was considered in [7] at M = 2 and M = 8, when there is no one-dimensional instability.

Confining ourselves to particle (ion) velocities much lower than the thermal electron velocity, we assume that the electrons have a Boltzmann distribution

$$n_e = n_0 \exp(e\varphi/T_e)$$

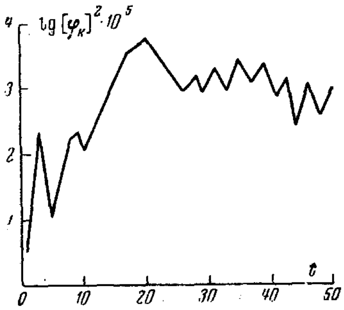

Fig. 1

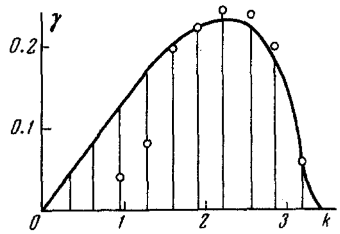

Fig. 2

It is then necessary to trace only the motion of the ions. In the one-dimensional case the problem reduces to a solution of the following equations:

$$\frac{\partial^2\varphi}{\partial x^2} = 4\pi e\,[n_0 \exp(e\varphi/T_e) - n_i], \qquad n_i = \int f_i(x, v, t)\,dv \tag{1}$$

$$\frac{dv_j}{dt} = \frac{e}{M_i}\int E(x)\,\rho_j(x)\,dx, \qquad E = -\frac{\partial\varphi}{\partial x}$$

$$\rho_j(x) = \rho_0 \exp\left\{-\left(\frac{x - x_j}{\sqrt{2}\,a}\right)^2\right\} \tag{2}$$

where φ is the potential, $f_i(x, v, t)$ is the ion distribution function, and $\rho_i(x)$ is the density of the j-th particle with halfwidth a in x space. In Eq. (2), the force acting on the j-th particle and its velocity are referred to the center of the particle.

When solving the boundary-value problem for the equation

$$\psi(\varphi) = \frac{\partial^2\varphi}{\partial x^2} + 4\pi e\,[n_i - n_0 \exp(e\varphi/\sqrt{T_e})] = 0$$

the latter was replaced by the equation (cf. [5])

$$\Psi(\varphi) = -\frac{\partial\Psi}{\partial\varphi}\frac{\partial\varphi}{\partial t}$$

the solution of which at $t \to \infty$ is the solution of Eq. (1). Periodic boundary conditions were imposed at the boundaries of the calculation interval.

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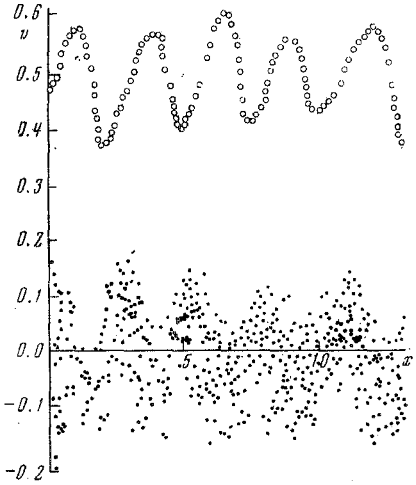

Fig. 3

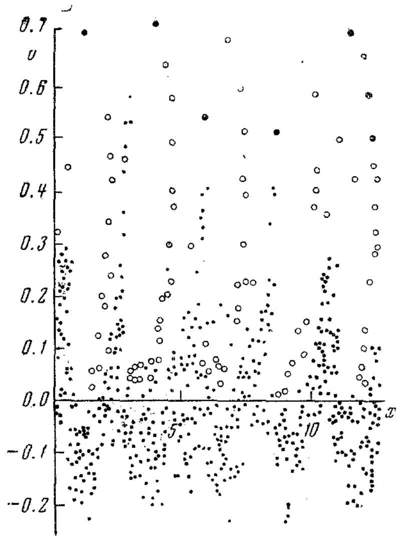

Fig. 4

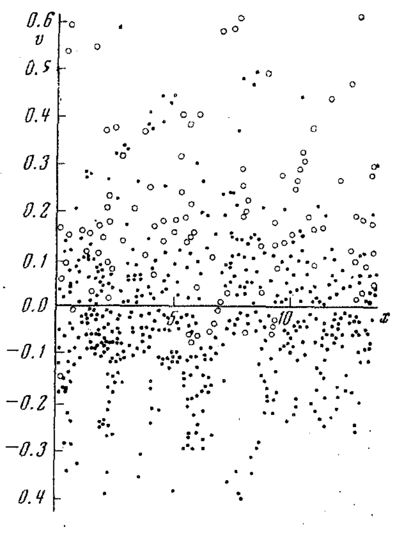

Fig. 5

The fitting method proposed in [5] was modified for the boundary conditions

$$\varphi(0) = \varphi(L), \quad \varphi'(0) = \varphi'(L) \tag{3}$$

We present below the results for the parameters:

$$\frac{n_b}{n_p} = \frac{1}{12}, \quad \frac{T_e}{T_i} = 10^{-2}, \quad u = 0.5c_*, \quad n_p D = 60$$

Here u is the beam velocity, n_b and n_p are the density of the beam and the plasma, respectively, and

$$c_* = \sqrt{T_e / M_i}$$

The number of particles is 2000, the length of the calculation interval is 20 Debye lengths.

Figure 1 shows the temporal evolution of the square of the amplitude of the Fourier harmonic of the electrostatic potential φ corresponding to the maximum increment (the time is measured in units of ω_0^{-1}). At first we have relaxation of the initial conditions $\varphi(x) \equiv 0$ to equilibrium thermal noise ($e\varphi \sim 5 \cdot 10^{-3} T_i$) – standing waves that are multiples of the calculation length. The beam is turned on at the instant t = 6 and two-stream instability develops after a certain residual relaxation. As seen from Fig. 1, the instability development can be subdivided into three stages.

The first stage is an exponential growth of the oscillation energy. Figure 2 shows the values of the increment $\gamma(k)$ calculated in accordance with the hydrodynamic model in the linear approximation [1] (solid curve) and the values obtained from the numerical experiment (circles). The thin vertical lines correspond to wavelengths that are multiples of the calculation length. As seen from Fig. 3, which shows the phase plane vx, the plasma and the beam are modulated at the instant t = 14 at the wavelength of the resonant harmonic. When $e\varphi_* \sim 0.05T_e$ (φ_* is the resonant potential), the linear stage terminates, and approximately 20% of the beam particles are accelerated to $v_{max}/u \sim 1.3$, while most of them (80%) are strongly decelerated to $v_{min}/u \sim 0.1$. After a time equal to several times ω_0^{-1}, saturation of the resonant harmonics sets in and the total energy of the oscillations is

$$\sum_k \int_0^L \left(\frac{E_k^2}{8\pi} + \frac{M_i v_{\sim k}^2}{2} \right) dx$$

where $v_{\sim k}$ is the oscillatory velocity of the plasma ions, and is of the order of the initial beam energy; the harmonics with wavelengths $\lambda \sim 2\lambda_*$ (λ_* is the resonant wavelength) continue to increase. There is no plateau on the distribution function, and characteristic "holes" are seen on the phase plane at t = 20 (Fig. 4), corresponding to concentration of the energy in three neighboring modes having a maximum increment.

The third stage is a fall-off in the oscillations and the onset of oscillations with a period

$$\tau^{-1} \sim \sqrt{e\varphi_{max} k^2 / M_i}$$

near a stationary level higher by more than one order of magnitude the level on the initial thermal noise. In this stage (t = 45), as follows from the phase picture (Fig. 5), the regular picture vanishes and the particles become mixed over the phase space. This allows us to assume that the influence of the beam is small after the resonant harmonics saturate, and the succeeding process is the damping of the large-amplitude wave for the case of a monochromatic wave, a case considered in [8].

To verify the accuracy of the calculation, the beam velocity was assumed to be $2c_*$. As expected, no instability was excited. Variation of the number of particles over the Debye length and over the length of the calculation interval did not cause noticeable changes in the results.

In [7], in the case of a weak interaction with $M = 8$, the energy of the oscillations begins to oscillate about a stationary level that exceeds by one order of magnitude the thermal noise, as in the conditions $n_b/n_p \ll 1$ under consideration. For $M = 2$ (strong interaction) the oscillation energy drops almost to the level of the thermal noise. The amplitudes of the Fourier harmonics are not given in [7], but the figure there reveals the existence of periodicity in phase space. Obviously, this is the result of the concentration of the oscillation energy in several adjacent modes.

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